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ABSTRACT

The conditional probability of marrying during the seven calendar months before or after a first birth, given that a women bears her first child in the first few years after initiating a consensual union and has not already converted the union into a marriage, is suggested here as a measure of the birth-connected marriage propensity. This propensity has fallen dramatically in data for Danish cohorts of women interviewed in 1975. The fall is partly a consequence of a strong decrease in nuptiality among unwed mothers.

CONTENTS

1. INTRODUCTION

When the increase in the prevalence of consensual unions became manifest in Scandinavia a decade ago, it was often hypothesized that such living arrangements largely were trial marriages, and that most unions would be converted into legal marriages no later than in connection with the first birth. When this failed to happen in many cases, the argument shifted to second births, and it was said that the latter would be the typical time to marry for those who stayed unmarried at the time of the first birth. See Bertelsen (1981, p. 2, Col. 2) for a statement of this nature concerning marriage at second birth in an interpretation of results from his investigation (Bertelsen, 1980) of the data from the 1975 Danish survey of family formation and female employment.

To investigate whether such impressions can stand up to empirical scrutiny, this paper shows how one can measure the propensity to marry in connection with a first birth in a consensual union, and demonstrates that such a propensity did indeed fall from some ninety percent to between a half and two thirds of cohabiting women in each cohort. This decrease took place between the cohorts born before and during the Second World War and the cohort born in 1951-55, according to the data Bertelsen also analyzed. Our propensity measure is the conditional probability of marrying in the seven calendar months just before or just after the birth, given that the woman has her first birth during (say) the first four years of a union and given that the union has not already been converted into a marriage. This probability is computed by extended life table methods in a simplified model where "disturbing" competing risks of union dissolution, divorce, wodowhood, and death have been eliminated. In this sense, our model probability is a "pure" measure of the interaction between natality and nuptiality.

Our computations are based on empirical rates of marriage and first birth to nulliparous women in consensual unions, on rates of first birth by marriage duration to women marrying at parity 0, and on rates of marriage following a first birth to cohabiting women. To the best of our knowledge, the latter rates have not been analyzed before to find the cohort effects and the influence of age at start of cohabitation. Despite the fact that our data set only has 158 women with cohabitational first births, analysis was possible by means of a proportional hazards model.

Only thirteen respondents reported cohabitational second births. Analysis of behavior following such second births in the Danish fertility survey has not been attempted here.

2. GENERAL MODEL

Our investigation of the extent to which cohabiting Danish women marry in connection with their first birth will be based on a simplified model whose state diagram is given in Figure 1. To bring out "pure" effects of natality and nuptiality, we disregard competing risks present in the various states, such as union dissolution for cohabiting women, divorce and widowhood for married women, second births to unmarried parity 1 women in consensual unions, and mortality for each subgroup. The model will be applied separately to each group of women, classified by cohort and by age at the start of cohabitation (for women in consensual unions) or at marriage (for married women). In our numerical application in later sections, women are grouped into birth cohorts born in 1926-40, 1941-45, 1946-50, and 1951-55, and the starting age groups

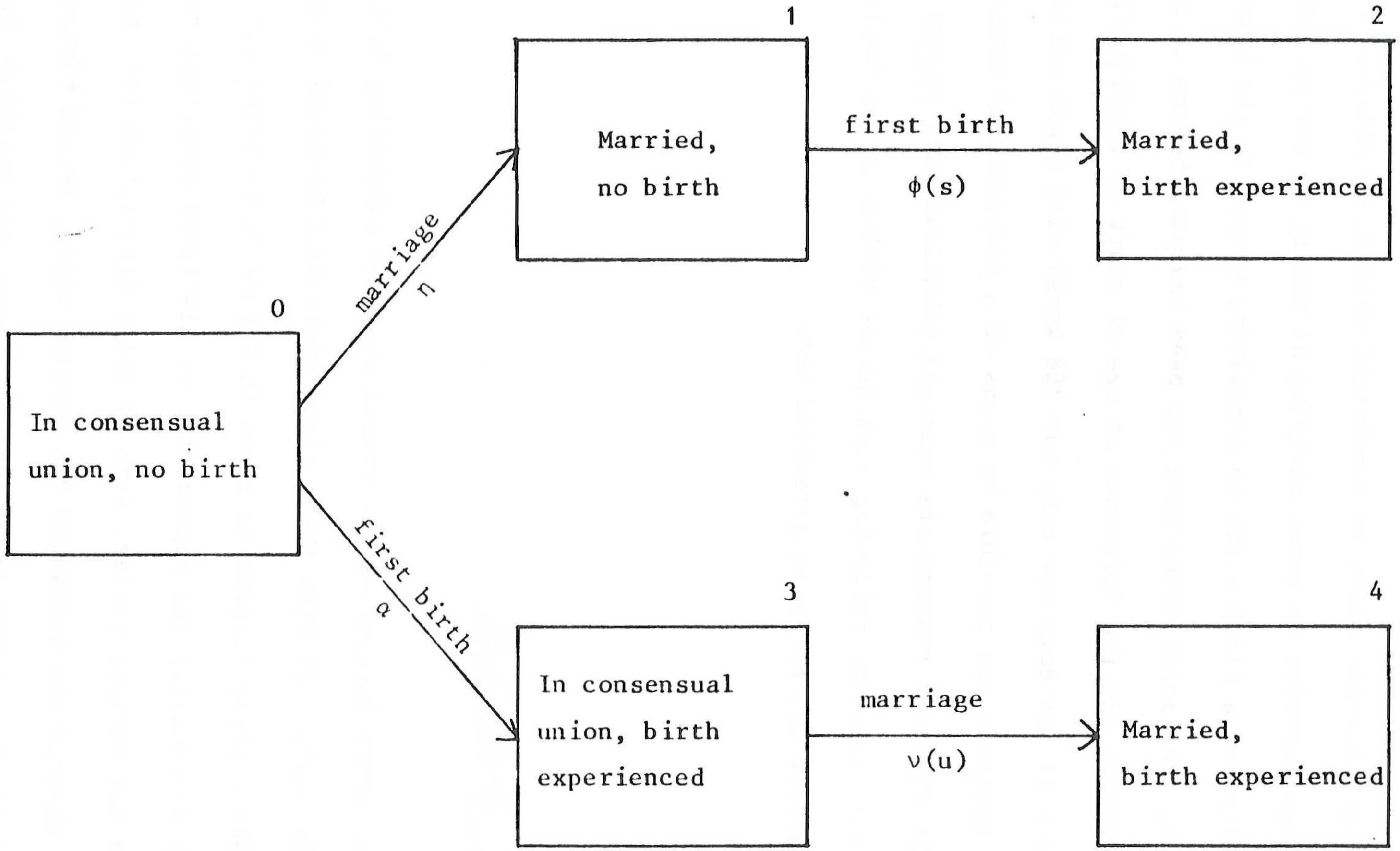

Figure 1. State diagram of basic model.

are 15-19 and 20-24 years.

Each box in Figure 1 corresponds to a demographic status, indicated by the text in the box. Marriages and first births are represented as transitions between corresponding boxes and are indicated by arrows in the figure. The intensity (or risk, or hazard) of making a given transition is a function named by a lower case Greek letter, indicated on the corresponding arrow in Figure 1. Their interpretation is as follows:

$\eta\, dt$ is the probability that a nulliparous woman who has been living in a consensual union for a time t, will marry her man by time t+dt.

$\alpha\, dt$ is the corresponding probability that she will have a first birth (instead) by time t+dt.

$\phi(s)\, ds$ is the probability that a nulliparous woman who has been married for a time s, will have a first birth by time s+ds.

$\nu(u)\, du$ is the probability that a cohabiting woman of parity 1 who had a first birth in the consensual union u time units ago, will marry before du more time units pass by.

In our computations, we take the marriage intensity η and the birth intensity α to be independent of union duration t. Except possibly for the first very few months of cohabitation, this is reasonable for the durations considered in our study population.

The marital birth intensity $\phi(s)$ is taken to be independent of the duration t of the premarital cohabitation but dependent on marital duration s. The marriage intensity $\nu(u)$ of parity 1 cohabiting women is taken as independent of union duration but dependent on time u since first birth.

The model states are numbered from 0 to 4 as indicated in the state diagram. We shall find a few model probabilities which are useful for our purposes. Let $p_j(\tau)$ be the probability that a nulliparous woman who has just started a consensual union (entered State 0), will be in State j at union duration τ. Then very standard arguments can be used to establish formulas like the following, where as always $\exp\{x\} = e^x$, and where, for simplicity, $\xi = \alpha+\eta$:

$$p_0(\tau) = \exp\{-\xi\tau\};$$

$$p_1(\tau) = \int_0^\tau \exp\{-\xi t\}\ \eta \exp\{-\int_0^{\tau-t} \phi(s)\,ds\}dt$$

$$= e^{-\xi\tau}\eta\zeta(\tau),$$

where

$$\zeta(\tau) = \int_0^\tau \exp\{\xi u - \int_0^u \phi(s)\,ds\}du;$$

and

$$p_2(\tau) = \int_0^\tau \exp\{-\xi t\}\ \eta[1-\exp\{-\int_0^{\tau-t} \phi(s)\}]dt$$

$$= m_0(\tau) - p_1(\tau),$$

where

$$m_0(\tau) = \int_0^\tau \exp\{-\xi t\}\ \eta\ dt = \frac{\eta}{\xi}(1-e^{-\xi\tau})$$

is the model probability that a newly-formed consensual union will have been transformed into a marriage before any first birth by duration τ. Similar formulas for $p_3(\tau)$ and $p_4(\tau)$ result from the simultaneous substitutions $\alpha\to\eta$, $\eta\to\alpha$, and $\phi(\cdot)\to\nu(\cdot)$.

Other probabilities can be derived from these formulas direct or by similar reasoning. For instance, the probability that our newly-baked female cohabitant has given birth by union duration τ is

$$\pi(\tau) = p_2(\tau) + p_3(\tau) + p_4(\tau) = 1 - p_0(\tau) - p_1(\tau),$$

and the model probability that she will marry and then have a first birth no later than (say) v time units after marriage, both by union duration τ, is

$$\beta_2(\tau) = \int_0^{\tau-v} \exp\{-\xi t\}\, \eta[1-\exp\{-\textstyle\int_0^v \phi(s)ds\}]\,dt$$

$$+ \int_{\tau-v}^{\tau} \exp\{-\xi t\}\, \eta[1-\exp\{-\int_0^{\tau-t} \phi(s)ds\}]\,dt.$$

If v is suitably small, say seven to eight months, then

$$\omega \equiv \exp\{-\textstyle\int_0^v \phi(s)ds\}$$

is essentially the probability that the woman did <u>not</u> carry a pre-marital pregnancy to term during the first months of her marriage. For the sake of illustration and because $\beta_2(\tau)$ will turn out to be of prime interest for our purposes, we indicate the following standard argument for its formula.

To achieve the composite event of marrying and subsequently giving birth before time τ and before the marriage has lasted for v time units, the newly-started cohabiting nulliparous woman must first remain unmarried at parity 0 in the union for some time t (between 0 and τ), she must then marry at time t (i.e., before time t+dt), and she must have left state 1 ("married, no birth") again before a further interval of v more time units, or before total time τ, whichever comes first. The corresponding sequence of probabilities are

$e^{-(\alpha+\eta)t}$	to remain in state 0 until time t,
$\eta\, dt$	to then move to state 1, and
$1-\omega$	to leave state 1 for state 2 before v more time units go by, if $0<t<\tau-v$.

If $\tau-v\leq t<\tau$, then the relevant final probability is $1-\exp\{-\int_0^{\tau-t}\phi(s)ds\}$ instead. Multiplying these together and adding over all suitable values of the time t of marriage then gives the formula for $\beta_2(\tau)$. It can easily be rewritten as

$$\beta_2(\tau) = m_0(\tau) - m_0(\tau-v)\omega - e^{-\xi(\tau-v)}\, p_1(v),$$

which can also be seen directly by decomposing the event of being married by union duration τ into four disjoint sub-eventsas follows:

1^o. The woman may remain unmarried until union duration $\tau-v$ and then marry but bear no child before duration τ.

2^o. Alternatively, she may remain unmarried until time $\tau-v$ but then both marry and bear a child before duration τ.

3^o. She may also marry before time $\tau-v$ and bear no child before marital duration v.

4^o. Finally she may marry before time $\tau-v$ and have a child before v time units of marriage.

Therefore, we get $m_0(\tau)$ by adding up the probability $\exp\{-\xi(\tau-v)\}\ p_1(v)$ of Event 1, the probability $m_0(\tau-v)\omega$ of Event 3, and the sum $\beta_2(\tau)$ of probabilities for Events 2 and 4. The above formula for $\beta_2(\tau)$ then results when the formula for $m_0(\tau)$ is reorganized.

Several further formulas connect our functions. Substantively the most interesting one is perhaps the relation

$$p_2(\tau) = \beta_2(\tau) +$$

$$\int_0^{\tau-v} e^{-\xi t}\, \eta[\exp\{-\int_0^v \phi(s)ds\} - \exp\{-\int_0^{\tau-t} \phi(s)ds\}]dt,$$

which follows easily from previous formulas. The difference $p_2(\tau) - \beta_2(\tau)$ is the model probability that our woman will marry and then bear a child more than v time units later, but before union duration τ.

We have used the subscript 2 in the symbol $\beta_2(\tau)$ because the woman ends up in State 2 by duration τ as an outcome of the composite event in question. The corresponding formulas for the model probability $\beta_4(\tau)$

that the newly-started nulliparous cohabiting woman will end up in State 4 no later than v time units after having given birth while still unmarried, follows from the formulas for $\beta_2(\tau)$, again by the simultaneous substitution $\alpha \to \eta$, $\eta \to \alpha$, $\phi(\cdot) \to \nu(\cdot)$.

Let us agree to say that a cohabiting woman <u>marries in connection with her first birth</u> if, given that she ever has a first birth, she also marries no more than v time units before or after that birth, where v is some figure like seven months. This means that, given that she does not stay on in states 0 or 1, she moves in the life paths through states $0 \to 1 \to 2$ or $0 \to 3 \to 4$ with transitions no more than v time units apart. Given that the first birth occurs by total time τ after initial cohabitation, the model probability that she is recorded to marry in connection with the birth by the same total time is

$$g(\tau) = \{\beta_2(\tau) + \beta_4(\tau)\}/\pi(\tau).$$

We use this conditional probability as a first index of the propensity to marry in connection with the first birth within a horizon of τ time units. It is a natural measure of such a propensity on several counts:

The very phrase "to marry in connection with the first birth" suggests that the marriage and the first birth should not enter symmetrically into the definition of the propensity measure, but that one should postulate (condition on) the occurrence of a birth. This does not in itself imply any direct underlying assumption of any causality between the marriage and the birth; one leaves open the possibility that both may have been caused by a common factor, say by a decision by the woman and the man to start building a more conventional family. Note that in reality, we also condition on the existence of a consensual union, and we restrict ourselves here to this case.

In times of changing natality, the conditioning on the occurrence of a first birth also helps to counterbalance possible unintended effects on a birth-connected marriage propensity index. If fertility falls, the unconditional probability $\{\beta_2(\tau) + \beta_4(\tau)\}$ of marrying and having a first birth, both within (say) seven months of each other, may also fall, irrespective of whether nuptiality changes. This consideration is of general interest in principle. In our own application, it is actually less important, for the birth intensities α and $\phi(\cdot)$ have not changed so much over the cohorts observed in our data.

The limitation to a finite horizon τ is practical for a number of reasons. It reminds us that the conditional probability of marrying in connection with a first birth does depend on the length of time which we allow for a birth to appear, a fact which is covered up in the initial verbal formulation. The dependence is quite important in practice.

By controlling τ, we also have some leverage on the realism of the model. If τ becomes too large, several model assumptions may become questionable: The constancy of the intensities η and α may become unreasonable, the assumed independence of all four intensities of actual age attained (beyond the control provided by the groupings by starting age) may be unrealistic, and so may other explicit or implicit assumptions involved. We could try to remedy some of these breakdowns by introducing more complex intensity functions, but we would then quickly run up against problems of model specification as well as of parameter estimation from available data.

If one is worried about nonmarital births, then the conditional model probability

$$h(\tau) = \{p_2(\tau) + \beta_4(\tau)\}/\pi(\tau)$$

that a first birth occurs in marriage or no later than v time units after marriage is perhaps at least as interesting as $g(\tau)$ is. So is probably the conditional model probability

$$\rho(\tau) = g(\tau)/\{1 - [h(\tau) - g(\tau)]\}$$

that a woman will marry in connection with her first birth, given that she has not done so before, when we still use a horizon of τ time units.

Let us call $\rho(\tau)$ <u>the birth-connected marriage probability</u> for horizon τ.

Our Section 5 presents empirical values for the probabilities $g(\tau)$, $h(\tau)$, and $\rho(\tau)$. Before that, Section 3 indicates how values for α, η, and $\phi(\cdot)$ were selected, and Section 4 has a novel empirical analysis of the post-natal marriage intensity $\nu(\cdot)$. The elementary and tedious detail of some further formula transformation needed for our numerical work is relegated to Appendix A.

3. MARRIAGES AND FIRST BIRTHS IN CONSENSUAL UNIONS, AND MARITAL FIRST BIRTHS

To illustrate our general theory, and because of its independent interest and our good knowledge of these data, we have selected values for our model parameters on the basis of the Danish fertility survey of 1975. The survey achieved interviews with 5 240 respondents, some 88 percent of its two-stage strafified random target sample of women of all marital statuses born in 1926-55. The sample has been described by Bertelsen (1980, Bilag 1), who has also given the questionnaire <u>in extenso</u> (Bilag 2). See Finnäs and Hoem (1980) for a description in English.

Marriages and first births recorded in these data for nulliparous women living in consensual unions have been studied by Hoem, Rennermalm, and Selmer (1984), who devised a method to adjust the corresponding computed vital rates for certain design biases due to the fact that the Danish questionnaire largely only obtained the latest consensual union (if any) before interview or before any latest marriage before interview. We found no important dependence on the achieved length of cohabitation in the empirical rates, computed according to the duration of the consensual union, month by month. Rate movements were dominated by random variation, except that the cohabitational marriage rates consistently rose somewhat during the first few months of the union. The marriage rates for women who reported that they started a consensual union at ages 15-19 were somewhat lower than corresponding rates with a reported starting age of 20-24, while first birth rates showed a strong dependence on starting age in the opposite direction. We found no perceptible trend in the first birth rates across cohorts. Table 1 lists the empirical rates used in calculations presented here. The marriage rates are for recorded ordinal months 5 to 23, combined; the birth rates are for ordinal months 1 to 23 of the consensual union. The calendar month in which the consensual union was recorded to start, counts as ordinal month 0, the next calendar month is ordinal month 1, and so on. The union is taken to have started in the middle of the month, on the average, so ordinal month 0 is regarded as half a month long. This makes ordinal month k last between durations $k-\frac{1}{2}$ and $k+\frac{1}{2}$ months, for $k\geq 1$. Our practice of deleting the first few ordinal months in the rates computed for Table 1 relieves us of the need to consider such details deeply at the present stage, but they enter in subsequent computations, so we may as well present our convention right away.

The rates in Table 1 have been used as values of our intensity parameters η (for marriages) and α (for first births) for durations from 0 through 48 ordinal months in all of our computations. The extension of these empirical rates down to duration 0 and up beyond a duration of 24 months will have given minor deviations from any results based on more complete data, but such deviations must be unimportant for our general empirical conclusions below (Section 5).

First (and second) births to <u>married</u> women in the same data have been studied by Hoem and Selmer (1984). According to their findings, it suf-

Table 1. Marriage rates and first birth rates to nulliparous Danish women in consensual unions, by cohort and by age at start of union. For Danish fertility survey of 1975, adjusted for design biases. Per 1 000 women per month.

Cohort born in	Starting ages 15-19	20-24
	Adjusted marriage rate[a)]	
1926-40	63	75
1941-45	61	57
1946-50	36	40
1951-55	15	17
	Adjusted rate of first birth[b)]	
1926-55	7.8	4.8

a) Empirical rates for ordinal months 5 to 23, combined.

b) Empirical rates for ordinal months 1 to 23, combined.

Table 2. First birth rates to married Danish women, by age at marriage and by duration of marriage. For cohorts in Danish fertility survey of 1975. Per 1 000 women per month.

Duration of marriage (ordinal months) a)	Ages 15-19 at marriage. Cohorts born in		Ages 20-24 at marriage, Cohorts born in
	1926-50	1951-55	1926-55
0	19	23	7
1	31	12	10
2	61	12	15
3	105	75	20
4	158	140	46
5	185	149	75
6	202	136	80
7	97	65	38
8	52	0	23
9	33	23	42
10	69	99	45
11	53	0	37
12	60	27	40
13-48	43		36

a) Ordinal month 0 covers marriage durations up to half a month. Ordinal month k goes from duration $k-\frac{1}{2}$ to $k+\frac{1}{2}$ months for $k \geq 1$.

fices for our purposes to use the marital first birth rates of Table 2, as we have done. There is appreciable duration dependence in these rates over the months in the first year of marriage, but we may as well use a constant first birth rate over the second to fourth year of marriage. Women married at ages 15 to 19 have a higher natality than respondents married at 20 to 24 years. Our youngest cohort, born in 1951-55, definitely has a lower proportion of pregnant brides than previous cohorts among those marrying at ages 15-19, but not for marriage ages 20-24, so we need to use separate birth rates during the first year of marriage, but only for the teenage brides of this cohort. Otherwise, we have not been able to detect important trends over our cohorts in marital first birth rates, and Table 2 contains rates for all (or most) cohorts combined. The main features of these considerations are evident in Figure 2.

It remains to specify values for the marriage intensity $\nu(\cdot)$. This cannot be done on the basis of previous investigations, so our next section contains an empirical analysis of marriages to one-child mothers in consensual unions.

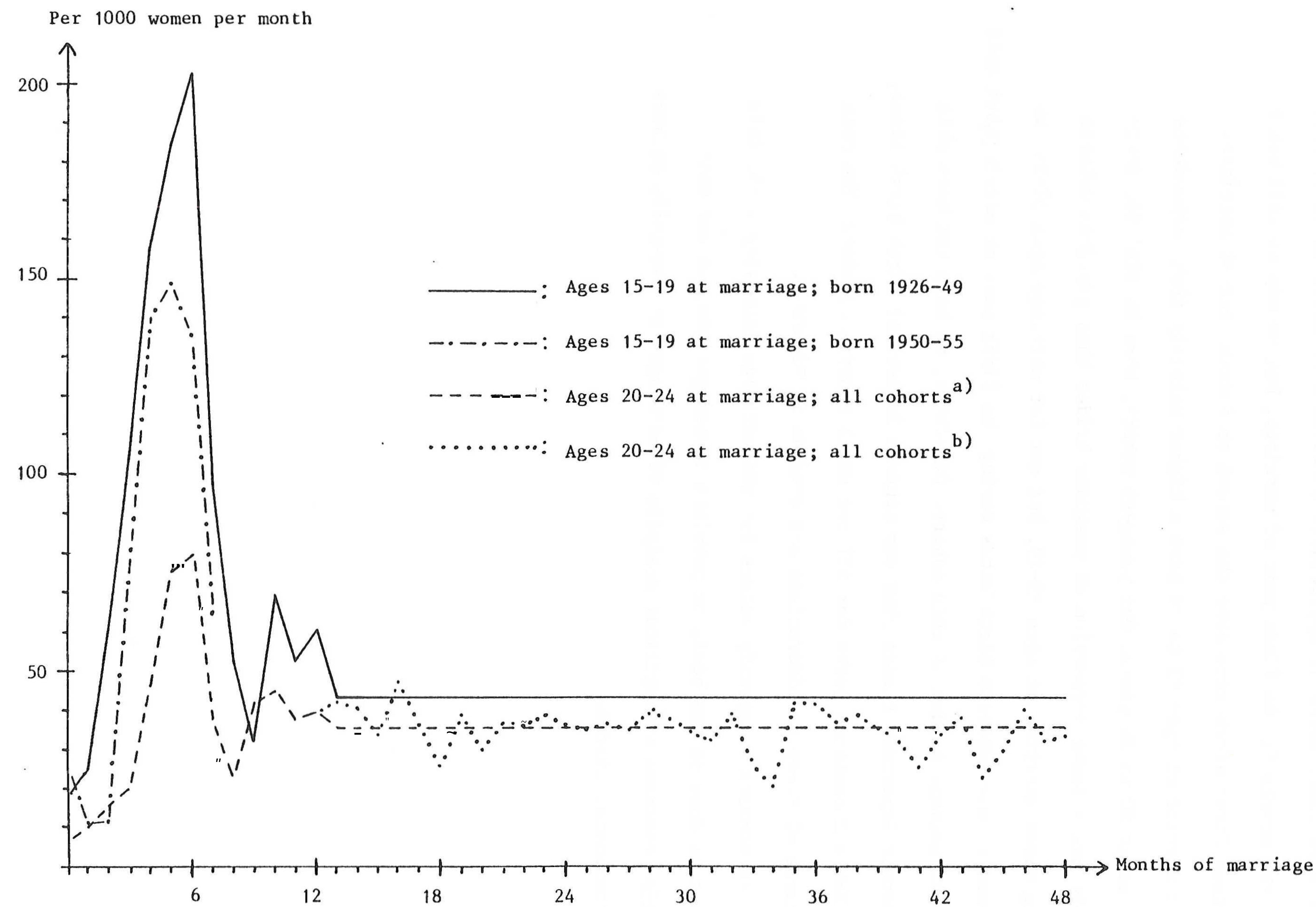

Figure 2. Occurrence/exposure rates for marital first births. Danish fertility survey of 1975. For ages 20-24 at marriage, the plotted rates are by single months of marriage up to month 12, inclusive, and then a) for months 13 to 48 combined, and b) also by single months 13 to 48.

4. MARRIAGE AMONG COHABITING MOTHERS

Among the 5 240 respondents in the Danish fertility survey of 1975, 158 women reported a first birth in a recorded consensual union, distributed over cohorts and over ages at reported start of cohabitation as in Table 3. Out of the 158, 102 had marriage to their cohabitant as their next subsequent reported vital event, 13 had a second nonmarital birth in the same consensual union, and the rest (43 women) had no further recorded vital event before interview. One can get some sensible results by using conventional multiple decrement life table methods to analyze 158 life history segments as if they came from a homogeneous population. On this basis, Hoem, Rennermalm, and Selmer (1984, Figure 3) have computed and plotted marriage rates for unmarried women in consensual unions, by duration since first birth, combined for all cohorts and for all ages 15-24 at reported start of cohabitation. The conventional approach would be to further analyze cohort effects and the influence of age at the start of cohabitation by partitioning the cases into subgroups and by separate analysis of each subgroup. Since we only have 158 life history segments available, such subgrouping would soon be counterproductive, however, and we have instead tackled the study of nuptiality among parity 1 women in consensual unions by specifying and analyzing a proportional hazards model. We have not even attempted to investigate the behavior of the thirteen two-child mothers in consensual unions after their second births.

Hazard models of this nature have been described in an excellent introduction by Trussell and Hammerslough (1983), to whom we refer for further description. It should suffice here to mention that we specify the relevant marriage intensity for ordinal month x after the first birth to a cohabiting woman in cohort k with starting age group s of

Table 3. Number of cohabiting women observed to experience a nonmarital first birth and possibly a subsequent marriage or nonmarital second birth, by cohort and by age at start of consensual union.

Cohort born in	Age at start of consensual union					
	15-19 years			20-24 years		
	Nonmarital first births	Subsequent marriages a)	Nonmarital second births b)	Nonmarital first births	Subsequent marriages a)	Nonmarital second births b)
1926-30	1	1	0	2	2	0
1931-35	4	4	0	6	3	3
1936-40	6	6	0	3	3	0
1941-45	23	18	5	15	13	1
1946-50	27	24	0	24	10	2
1951-55	34	15	2	13	3	0
Sum	95	68	7	63	34	6

a) Numbers married before interview, after a first birth and before any second birth.

b) Numbers with a second birth in the same consensual union, before interview. Out of the thirteen cases recorded here, only three were still unmarried at interview.

cohabitation as the product

$$\nu_{ksx} \equiv a_x b_k c_s,$$

for x=0,1,...; k=1,2,3,4; and s=1,2. In practice, x goes up to ordinal month 16 inclusive, after which our data peter out. The cohorts are k=1 for women born in 1926-40, k=2 for women born in 1941-45, k=3 for 1946-50, and k=4 for those born in 1951-55. As before, the starting age groups are s=1 for ages 15-19, and s=2 for ages 20-24 at the beginning of the recorded consensual union. We have combined our three oldest five-year cohorts into the fifteen-year cohort born in 1926-40 to achieve a group of a resonable size (compare Table 3). We have got substantially the same results as those reported presently from a parallel analysis where the diminutive groups of three women altogether born in 1926-30 were left out, and where women born in 1931-35 were kept separate from those born in 1936-40.

No marriages were reported in our data in the same month as the first birth (i.e., in ordinal month 0 since first birth). This has saved us from having to face the problem that we would not know which of the two events occurred first in the same month. The occurrences and exposures of the 4×2×16=128 other combinations of four cohorts, two starting age groups, and sixteen ordinal months of observation available have been tabulated in Appendix E for the benefit of those who would like to carry out their own experiments. Note that only 51 of the occurrences are nonzero, and that only a single one of the positive occurrences is as large as 4.

The multiplicative three-factor model for ν_{ksx} fits our data nicely. (See Appendix B.) The numerical job was done by means of an efficient program called LOGLIN (see Appendix C), developed by Olivier and Neff (1976) and recommended before by Trussell and Hammerslough (1983, Appendix A) and others. We have chosen to normalize the model parameters (and secure their identification) by setting $b_2=c_1=1$. Table 4 contains the LOGLIN maximum likelihood estimates of our $\{a_x\}$, $\{b_k\}$, and $\{c_s\}$. They can be interpreted as follows.

In the situation studied here (State 3 of Figure 1), women who were born in 1941-45 and who reported an age of 15-19 years at the start of their consensual union, have a marriage intensity of $\nu_{2,1,x}=a_x$ in ordinal month x after first birth. Estimates $\{\hat{a}_x\}$ of the $\{a_x\}$ have been listed in Panel A of Table 4. A plot of the $\{\hat{a}_x\}$ is almost indistinguishable from that of Figure 3 of Hoem, Rennermalm and Selmer (1984), except that the ordinate axis has now been rescaled. The ordinates of their figure appear as those of our Table 4 (Panel A), all largely multiplied by some average of products $\hat{b}_k\hat{c}_s$ of the estimates $\hat{b}_k$ and $\hat{c}_s$ of Panels B and C in Table 4.

Now turn to a woman born in 1946-50 who started to live in a consensual union at age 15-19 years and who had a cohabitational first birth in the union. In each subsequent month, she had a constant sub-"risk" of marriage estimated as 100(1-0.770)=33 percent of the corresponding hazard for her counterpart born five years earlier, for her marriage intensity in ordinal month x since first birth would be $\nu_{3,1,x}=b_3a_x$. If the same woman, born in 1946-50, had a reported starting age of 20-24 years instead, then her nuptiality in ordinal month x would be $\nu_{3,2,x}=c_2b_3a_x$, and her constant sub-"risk" as compared to a sister born in 1941-45 starting at age 15-19 years, would be an estimated $100(1-0.770\cdot0.600)=53.8$ percent.

The estimated duration structure $\{\hat{a}_x\}$ shows a greatly inflated nuptiality in the first months following a first cohabitational birth. In this sense at least, cohabiting Danish women certainly have married to a considerable extent in connection with a first birth. Our computations on the birth-connected marriage propensity $\rho(\tau)$, reported in our next section, have used the estimates in Table 4, extended for durations

Table 4. Parameter estimates for the multiplicative model of nuptiality after a first birth to Danish women in consensual unions, by age at start of union and by time since first birth. For cohorts in Danish fertility survey of 1975.

A. Basic structure of time dependence[a)]

Ordinal month	Rate per 1000 per month	Ordinal month	Rate per 1000 per month	Ordinal month	Rate per 1000 per month
0	0	6	50.2	12	94.8
1	61.0	7	70.6	13	102.2
2	216.5	8	56.1	14	54.9
3	185.6	9	79.7	15	59.4
4	58.2	10	64.1	16	31.7
5	109.5	11	45.2		

B. Cohort level[b)]

Cohort born in	Factor
1926-40	0.933
1941-45	1
1946-50	0.770
1951-55	0.493

C. Starting age factor[c)]

Age at start of union	Factor
15-19	1
20-24	0.600

a) Estimates $\{\hat{a}_x\}$ of model parameters $\{a_x\}$.

b) Estimates $\{\hat{b}_k\}$.

c) Estimates $\{\approx{c}_s\}$.

(since first birth) above 16 months by letting a_x=0 for x>16.

Panel C in Table 4 shows that an age of 20-24 at the start of the consensual union reduces the nuptiality in each group by some forty percent. We have tried a few further groupings and analyses of subsets of these data (not reported here), and have found such hazard reduction to be a robust feature of the outcomes. This finding goes counter to any interpretation to the effect that women who started a consensual union at ages of 20-24 years were more prone than those who started at ages 15-19 to act as if their union were trial marriages, to be converted into legal marriages no later than just after the first birth. One might perhaps hypothesize instead that women who started co-habiting as teenagers, were more inclined to legalize their union after first birth because they, more than their somewhat older sisters starting cohabitation at ages 20-24, felt a need for the support of a more conventional family pattern. It seems more plausible, however, to seek an explanation for these Danish respondents in the selectivity of the cohabitation process. As unmarried cohabitation became a feasible and accepted lifestyle, women with a strong family orientation will have started cohabiting early and then married as soon as "the time was ripe", which was felt to be more quickly for them than for less family oriented women.

This selective behavior will have been reinforced by the fact that those who started cohabiting at ages 20-24, did so on the average five more calendar years along the path of increasing popularity of nonmarital living arragements.

According to Panel B in Table 4, postnatal cohabitational nuptiality fell by a quarter from the cohort born in 1941-45 to the one from 1946-50, and then again by another quarter of its base level, so that the cohort born in 1951-55 had only half the nuptiality of the World War II cohort. According to the table values, the nuptiality level actually increased from the pre-War cohort to the one born in 1941-45. This feature is robust against a few experiments with different cohort selections (not otherwise reported here). If this reflects a real increase, our best guess for an explanation would again be a selectivity hypothesis. It is possible that Danish women who bore children in consensual unions in our oldest cohorts were representatives of lifestyles which were rather unorthodox at the time. This could have been manifested also in somewhat lower nuptiality after a first birth than in the War cohort of women, whose order one births appeared at a time when nonmarital cohabitation had become more common, but where marrying was still the regular thing to do when a pregnancy appeared or had just been completed (if not before). In later cohorts, nonmarital childbearing then became progressively more accepted and more prevalent.

We are not sure that the lower value of the cohort factor for women born in 1926-40 does reflect a real phenomenon, however, for it is not significantly smaller than the factor for the cohort of 1941-45, according to the relevant likelihood ratio test. (The test statistic is 0.58, on a single degree of freedom in the chi-square distribution. See Appendix D.) We have kept the estimated cohort factors of Table 4 in the computations reported below nevertheless, mostly because we have wanted to show how little a cohort factor of 0.933 (instead of a factor of 1) influences the outcome.

It may have some interest to convert the parameter values of Table 4 to the domain of probabilities. If

$$q = 1 - \exp\{-\int_0^t \nu(u)du\},$$

then q is the probability that a woman who has just given a first birth in a consensual union, will marry her man no later than t time units later, computed by the single decrement life table method. Table 5 shows that when t is chosen as the end of the sixteenth calendar month following the birth (in effect $t=16.5$), then this probability has fallen from some three fourths to about a half over our cohorts for women starting their consensual union as teenagers. For starting ages 20-24, the probability was as small as a half in our pre-War cohorts already, and it has fallen to about one third by our youngest cohort. Danish women with cohabitational first births have certainly married, but they have not rushed to do so.

Table 5. Partial probability[a)] of marrying no later than sixteen calendar months after first birth in consensual union. By cohort and by age at start of union. Percent.

Cohort born in	Age at start of union	
	15-19	20-24
1926-40	71.3	52.8
1941-45	73.8	55.2
1946-50	64.4	46.1
1951-55	48.3	32.7

a) Computed by the single decrement life table method.

5. RESULTS ON MARRIAGES CONNECTED WITH FIRST BIRTHS

Armed with the model of Section 2, the parameter values presented in Sections 3 and 4, and the computational formulas of Appendix A, we have calculated values for the birth-related marriage probabilities $g(\tau)$, $h(\tau)$, and $\rho(\tau)$, for horizons τ of 12, 24, 36, and 48 months, for each of the four cohorts of our analysis. Table 6 contains such values of $g(\cdot)$ based on rates for women who started cohabiting at ages 15-19 and who also married in the same age bracket (if ever). It also contains similar values based on rates for cohabitational starting ages 20-24 and the same ages at marriage. Of course, a woman who starts cohabiting in one five-year age bracket, may marry in a subsequent age bracket, and we could have made similar calculations for women who started cohabiting at ages 15-19 and who married at ages 20-24, if ever. We do not report such outcomes here, however, for we doubt that the extra insight gathered would be worth the effort.

Tables 7 and 8 contain similar values for $h(\cdot)$ and $\rho(\cdot)$. All tables indicate that the choice of horizon τ can be quite important for the outcome. By a union duration of four years, the probabilities largely seem to have stabilized, and our further comments concentrate on τ=48 months. A comparison of the results tabulated for the cohorts born in 1926-40 with those for the cohort from 1941-45 suggests that the effect of using a cohort factor of 0.933 (for 1926-40) instead of a factor of 1 is rather unimportant.

The final column of Table 7 shows that as many as ninety-odd percent of the first births to originally cohabiting women born in 1926-45 appeared in marriage or shortly after marriage, according to our model calculations, counting all births during the first four years of a union. In this respect, their consensual unions were indeed trial marriages. By our youngest cohort, born in 1951-55, this had fallen to some seventy-

Table 6. Conditional model probability $g(\tau)$ of marriage in connection with a given first birth to a Danish woman during a union of specified duration. For selected durations, by starting age of cohabittion and by cohort. Percent.

Cohort born in	Duration in months 12	24	36	48
	Starting ages 15-19			
1926-40	83.0	75.0	68.4	63.9
1941-45	83.2	75.3	68.7	64.3
1946-50	74.5	70.8	66.1	62.6
1951-55	50.3	52.3	49.8	47.5
	Starting ages 20-24			
1926-40	72.1	51.7	40.5	34.5
1941-45	69.1	52.0	41.6	35.7
1946-50	62.0	49.9	41.1	35.8
1951-55	43.1	40.7	36.3	33.0

Table 7. Conditional model probability $h(\tau)$ of being married at the time of first childbearing or of marrying no later than seven calendar months afterwards for a Danish woman with a union of specified duration. For selected durations, by starting age of cohabitation and by cohort. Percent.

Cohort born in	Duration in months 12	24	36	48
	Starting ages 15-19			
1926-40	87.0	91.7	93.0	93.7
1941-45	87.1	91.8	93.1	93.8
1946-50	77.6	84.6	86.9	88.1
1951-55	51.8	62.7	67.4	70.2
	Starting ages 20-24			
1926-40	82.8	91.1	93.5	94.6
1941-45	78.7	88.5	91.6	93.0
1946-50	70.1	82.5	86.8	88.9
1951-55	48.0	63.6	70.2	73.9

Table 8. Conditional model probability $\rho(\tau)$ of marriage in connection with a first birth to a Danish woman during a consensual union of a specified duration, given no previous conversion of the union into a marriage. For selected durations, by starting age of cohabitation and by cohort. Percent.

Cohort born in	Duration in months 12	24	36	48
	Starting ages 15-19			
1926-40	86.5	90.0	90.7	91.0
1941-45	86.6	90.2	90.9	91.2
1946-50	76.9	82.2	83.5	84.0
1951-55	51.1	58.4	60.5	61.4
	Starting ages 20-24			
1926-40	80.7	85.3	86.2	86.5
1941-45	76.4	81.9	83.2	83.6
1946-50	67.4	74.1	75.7	76.4
1951-55	45.3	52.8	54.9	55.8

odd percent, so that a bit more than every fourth first birth occurred in the consensual union, with no connected marriage following shortly afterwards.

According the the final column of Table 6, a much larger percentage of these marriages occurred in connection with the first birth among the very young women than among those who started cohabiting at ages 20-24. The latter married more at an earlier stage of the union. For them the model probability of a birth-connected marriage stayed stably around a third, for the young women it declined from about two thirds to a half. These are the probabilities as seen from the perspective of a woman who had just started a consensual union. If we turn now to an expecting or just recent mother, the final column of Table 8 shows that she had a very high model probability of turning her union into a marriage in connection with her first birth if she had not done so before in our early cohorts. By the cohort born in 1951-55, this probability had fallen considerably, but it was still a good deal above one half. Cohabiting women continued to be quite prone to marry in connection with their first birth if they had not done so before, though some four out of every ten women had then come to to stay unmarried beyond the first seven month after their first birth.

Note that there is no assumption about a selection process due to population heterogeneity involved in these computations. No account has been taken of the possibility that women who stayed unmarried until about the time of their first birth (or beyond) may have had properties different from those who married at an earlier stage, apart from manifest behavior. The conditional model probabilities of Tables 6 to 8 have been computed on the possibly counterfactual assumption that all women had probabilistic properties completely determined by their own birth cohort and by their age at entry into a consensual union and subsequently into marriage. It is not population heterogeneity which makes each probability in Table 8 larger than its counterpart in Table 6; this relationship is caused solely by the fact that the probabilities of Table 8 are based on behavior up to a later stage in life than are those of Table 6, with the ensuing extra information about the woman.

APPENDIX A. FORMULAS FOR COMPUTATIONS

This appendix converts the formulas of our paper into a form more amenable to numerical computation. Let us describe a marriage as occurring in connection with a first birth if both events occur in the same calendar month or if the marriage occurs in one of the m calendar months before or after the birth. In our computations, m=7 months. Since, on average, an event occurs in the middle of the reported calendar month, this means that $v=m+\frac{1}{2}$ in the formulas of Section 2.

On similar reasoning, an interview is taken in the middle of the interview month on average. The horizon τ of the fomulas will be 12, 24, 36, or 48 months in our computations, in mimicry of interviews taken in the same month of the year as the start of cohabitation, but one, two, three, or four years later. In our computations, therefore, τ is regarded as an exact duration of 12, 24, 36, or 48 months. Our time unit is a month throughout the computations.

We assume $\phi(s)$ to be a constant in each ordinal month following marriage. By our definition of ordinal month 0 of marriage to be marriage durations from 0 to 0.5, ordinal month 1 to be the durations from 0.5 to 1.5 time units, and so on, this means that

$$\phi(s) = \begin{cases} \phi_0 & \text{for } 0 \le s \le \frac{1}{2}, \\ \phi_k & \text{for } k-\frac{1}{2} \le s < k+\frac{1}{2}, \quad \text{for } k \ge 1. \end{cases}$$

It is useful to let $s_0 \equiv \frac{1}{2}\phi_0$, $s_k = \frac{1}{2}\phi_0 + \phi_1+\ldots+\phi_k$ for $k \ge 1$.

To compute $\zeta(t)$, define

$\lambda_k(\delta)=\exp\{\xi(k-\frac{1}{2})-s_{k-1}\}[\exp\{(\xi-\phi_k)\delta\}-1]/(\xi-\phi_k)$ for $0<\delta\le 1$ and $k\ge 1$, while

$$\lambda_0 = (e^{\frac{1}{2}(\xi-\phi_0)} - 1)/(\xi-\phi_0).$$

Note that for integer $\ell \geq 1$ and for $\ell - 1 < u \leq \ell + \frac{1}{2}$,

$$\int_0^u \phi(s)ds = \int_0^{\frac{1}{2}} \phi(s)ds + \sum_{k=1}^{\ell-1} \int_{k-\frac{1}{2}}^{k+\frac{1}{2}} \phi(s)ds + \int_{\ell-\frac{1}{2}}^{u} \phi(s)ds$$

$$= \tfrac{1}{2}\phi_0 + \sum_{k=1}^{\ell-1} \phi_k + \phi_\ell(u-\ell+\tfrac{1}{2}) = s_{\ell-1} + \phi_\ell(u-\ell+\tfrac{1}{2}),$$

while for $0 \leq u \leq \frac{1}{2}$,

$$\int_0^u \phi(s)ds = \phi_0 u.$$

Therefore, if ℓ is a positive integer and if $0 \leq \delta \leq \frac{1}{2}$, then

$$\zeta(\ell+\delta) = \int_0^{\ell+\delta} \exp\{\xi u - \int_0^u \phi(s)ds\}du$$

$$= \int_0^{\frac{1}{2}} \exp\{\xi u - \phi_0 u\}du$$

$$+ \sum_{k=1}^{\ell-1} \int_{k-\frac{1}{2}}^{k+\frac{1}{2}} \exp\{\xi u - s_{k-1} - \phi_k(u-k+\tfrac{1}{2})\}du$$

$$+ \int_{\ell-\frac{1}{2}}^{\ell+\delta} \exp\{\xi u - s_{\ell-1} - \phi_\ell(u-\ell+\tfrac{1}{2})\}du,$$

or

$$\zeta(\ell+\delta) = \lambda_0 + \sum_{k=1}^{\ell-1} \lambda_k(1) + \lambda_\ell(\delta+\tfrac{1}{2}).$$

Since $v=m+\frac{1}{2}$ and since τ is an integer, inspection of the formulas of Section 2 shows that we need to compute $\zeta(m+\frac{1}{2})$, $\zeta(\tau)$, and $\omega = \exp\{-s_m\}$.

The value of the integral in the exponent of the formula for q at the end of Section 4 has been computed in a manner similar to that for $\int_0^u \phi(s)ds$ above. For cohort k and starting age group s, with t=16.5,

$$\int_0^{16.5} \nu_{ks}(u)du = \int_0^{16.5} a(u)\, b_k c_s du = b_k c_s \sum_{x=1}^{16} a_x.$$

Since $a_0=0$, an additional item of $\frac{1}{2}a_0$ disappears in the sum over $\{a_x\}$.

APPENDIX B. GOODNESS OF FIT TESTS

This appendix shows how one can test the fit of the multiplicative hazard model in Section 2, where the marriage intensity is

$$\nu_{ksx} \equiv a_x b_k c_s, \tag{B.1}$$

whith $b_2=c_1=1$. Consider women of cohort k in age group s at the start of a consensual union, who have a first birth while unmarried and while still living in that union. Let $M_{ks}(x)$ be the number of marriages recorded in our data in ordinal month x after the birth, and let $E_{ks}(x)$ be the corresponding number of woman-months of exposure. Our values of these quantities can be found in Tables 9 and 10 in Appendix E. The log-likelihood is

$$\ln \Lambda = \Sigma_k \Sigma_s \Sigma_x \{M_{ks}(x) \ln \nu_{ksx} - E_{ks}(x)\, \nu_{ksx}\}. \tag{B.2}$$

If no other restriction than nonnegativity is imposed on the $\{\nu_{ksx}\}$, then $\ln \Lambda$ is maximized by the occurrence/exposure rates

$$\hat{\nu}_{ksx} = M_{ks}(x)/E_{ks}(x), \tag{B.3}$$

and the maximal value $\ln \hat{\Lambda}$ of $\ln \Lambda$ results from replacing each ν_{ksx} by $\hat{\nu}_{ksx}$.

Under the multiplicative model (B.1), the maximal value of $\ln \Lambda$ becomes a somewhat smaller quantity $\ln \hat{\Lambda}_0$. If each $M_{ks}(x)$ is sufficiently large, the likelihood ratio test statistic

$$Q_0 = -2 \ln(\hat{\Lambda}_0/\hat{\Lambda})$$

is approximately chi-square distributed when the multiplicative model is true, with a number of degrees of freedom equal to KSA $-(A+K+S-2)$, where K is the number of cohorts, S the number of starting age groups, and A the effective number of duration intervals involved. In our case, K=4, S=2, and A=16. If some $E_{ks}(x)$ are zero, then the degrees of freedom is reduced by the number of zero exposures, provided each $M_{ks}(x)$

corresponding to a nonzero exposure is sufficiently large. In either circumstance, Q_0 can be used to test the fit of the multiplicative model. The program LOGLIN will automatically produce the current value of Q_0 and its degrees of freedom.

If a zero (or even a very small nonzero) occurrence $M_{ks}(x)$ appears for a nonzero exposure $E_{ks}(x)$, trouble arises. For $M_{ks}(x)=0$, (B.3) gives $\hat{\nu}_{ksx}=0$, and $\ln \hat{\Lambda}$ may be computed with $M_{ks}(x) \ln \hat{\nu}_{ksx} = E_{ks}(x)\hat{\nu}_{ksx}=0$, for we interpret 0 ln 0 as zero. LOGLIN makes this computation, reduces the degrees of freedom by one for each such zero occurrence, and prints a warning that the degrees of freedom may be wrong. In fact, the entire assumption that the chi-square distribution is appropriate for the test criterion may be wrong. To the best of our knowledge, one does not know what distribution Q_0 has when a zero (or a very small) occurrence appears in connection with a nonzero exposure. The program LOGLIN cannot have been developed for such a case; its great usefulness for life history analysis was probably understood only after it had been written.

As seems faily well known already, the LOGLIN program can be used in a different manner to produce a goodness of fit test to replace Q_0, as follows. Extend the multiplicative model $\nu_{ksx}=a_x b_k c_s$ by including interaction terms, so that, say,

$$\nu_{ksx} = a_x b_k c_s d_{kx} e_{ks} f_{sx}. \tag{B.4}$$

Here, $b_2=c_1=1$, as before, and also $e_{21}=1$ and $f_{1x}=d_{2x}=1$ for all x. Use LOGLIN to fit this model with interaction terms to the data, and call the corresponding maximal likelihood $\ln \tilde{\Lambda}$, say. The program will then automatically produce another likelihood ratio test statistic

$$\tilde{Q} = -2 \ln(\tilde{\Lambda}/\hat{\Lambda})$$

with an associated $\tilde{f}$ degrees of freedom, which can be compared to the above statistic Q_0 and to its f_0 degrees of freedom. Under the hypothesis that all interaction terms are redundant ($d_{kx} \equiv e_{ks} \equiv f_{sx} \equiv 1$), the difference

$$Q = Q_0 - \tilde{Q} = -2 \ln(\hat{\Lambda}_0/\tilde{\Lambda})$$

appears to be approximately chi-square distributed with $f=f_0-\tilde{f}$ degrees of freedom even in circumstances where Q_0 is not. There seems to be little hard knowledge about what those circumstances really are, so great care should be used in applying Q as a test statistic for the goodness of fit of the multiplicative model. It is our experience, however, that it often works well in practice, at least as a general indicator, even when Q_0 fails to give sensible results.

With the data of our Section 4, $Q_0=98.43$, $f_0=108$, $\tilde{Q}=31.72$, and $\tilde{f}=17$, which makes $Q=66.71$ on $f=91$ degrees of freedom, which formally implies nonrejection of the multiplicative model by a safe margin. The malady in the chi-square approximations to the distributions of Q_0 and $\tilde{Q}$ under their respective models (whose goodness of fit they ought to test) shows up in our case when Q_0 implies nonrejection of the simple multiplicative model, again by a safe margin, while conversely $\tilde{Q}$ strongly suggests that the more flexible model with two-way interactions be rejected and replaced by an even more complex model. (As computed formally, the rejection probability for $\tilde{Q}$ is $p=0.016$.)

APPENDIX C. LOGLIN'S MODEL VERSION

The program LOGLIN can be used to fit a model of the form

$$\nu_{ksx} = \exp\{\Delta+\alpha_x+\beta_k+\gamma_s\}$$

to the group- and duration-specific occurrences and exposures of the raw data (Appendix E). Here, $\Sigma_x\alpha_x = \Sigma_k\beta_k = \Sigma_s c_s = 0$. This model is equivalent to the multiplicative specification in (A.1). The parameters of the two formulations are related to each other (when we have chosen to let $b_2=c_1=1$) via the transformations

$$a_x = \exp\{\Delta+\alpha_x+\beta_2+\gamma_1\},$$

$$b_k = \exp\{\beta_k-\beta_2\},\ c_s = \exp\{\gamma_s-\gamma_1\}.$$

A model with interaction terms is transformed in a similar manner.

APPENDIX D. TEST FOR THE EQUALITY OF TWO COHORT FACTORS

Now <u>assume</u> that $\nu_{ks}(x) = a_x b_k c_s$, with $b_2=c_1=1$, and suppose that we want to test whether $b_1=1$ versus $b_1\neq1$, as in our Section 4. We keep the notation of Appendix B, and let $\hat{a}_x$, $\hat{b}_k$, and $\hat{c}_s$ be the maximum likelihood estimators of a_x, b_k, and c_s, respectively, when $b_2=c_1=1$ while b_1 is allowed to vary freely. Let the corresponding estimators be a_x^*, b_k^*, and c_s^* when b_1 is fixed at 1. (Of course, $\hat{b}_2=\hat{c}_1=b_1^*=b_2^*=c_1^*=1$.) The values of the latter estimators will be produced by LOGLIN if we combine the occurrences and the exposures separately for each starting age group in Cohorts 1 and 2, keep the data for Cohorts 3 and 4 unchanged, and then refit the three-factor multiplicative model to the combined data. Let $\ln \Lambda_0^*$ be the maximal log-likelihood under the new fit.

Then

$$Q_0^* = -2 \ln(\Lambda_0^*/\hat{\Lambda}_0)$$

will be approximately chi-square distributed on a single degree of freedom when $b_1=1$, and Q_0^* can be used as a test statistic for the hypothesis that $b_1=1$.

With our data, $Q_0^*=0.58$, so we get formal nonrejection of this hypothesis by a wide margin.

As noted in Appendix B, the program LOGLIN automatically provides the value $Q_0=98.43$ with $f_0=108$ formal degrees of freedom when it finds the estimates $\hat{a}_x$, $\hat{b}_k$, and $\hat{c}_s$. It also provides a value $Q^*=79.41$ on $f^*=77$ degrees of freedom as a formal likelihood ratio goodness of fit test statistic when it finds the estimates a_x^*, b_k^*, c_s^*. At first blush, one might perhaps believe that Q_0 and Q^* can be used to test whether $b_1=1$ in the same manner as $Q_0-\tilde{Q}$ was used to test the redundance of the interaction terms in Appendix B. This is an error easily made. If this were possible, then the proper difference to compute would be Q^*-Q_0 and the degrees of freedom would be f^*-f_0. However, with our data, both differences are negative ($Q^*-Q_0=-19.02$ and $f^*-f_0=-31$) and far from Q_0^* and 1 in absolute values, which shows that they cannot be used as indicated. The suggestion that Q^*-Q_0 be used as a test statistic is based on the impression that Q^* would equal $-2\ln(\Lambda_0^*/\hat{\Lambda})$, but it does not. Instead, $Q^*=-2\ln(\Lambda_0^*/\Lambda^*)$, where $\ln\Lambda^*$ is computed from the reorganized occurrences and exposures as follows:

Just as we defined $\hat{\nu}_{ksx}=M_{ks}(x)/E_{ks}(x)$ in Appendix B, we now also introduce

$$\nu_{2sx}^* = \{M_{1s}(x) + M_{2s}(x)\}/\{E_{1s}(x) + E_{2s}(x)\},$$

and $\nu_{ksx}^* = \hat{\nu}_{ksx}$ for $k=3,4$. Then

$$\ln \hat{\Lambda} = \sum_{k=1}^{4} \sum_{s=1}^{2} \sum_{x=1}^{16} \{M_{ks}(x) \ln \hat{\nu}_{ksx} - E_{ks}(x) \hat{\nu}_{ksx}\}$$

and

$$\ln \Lambda^* = \sum_{k=3}^{4} \sum_{s=1}^{2} \sum_{x=1}^{16} \{M_{ks}(x) \ln \nu^*_{ksx} - E_{ks}(x) \nu^*_{ksx}\}$$

$$+ \sum_{s=1}^{2} \sum_{x=1}^{16} \{[M_{1s}(x) + M_{2s}(x)] \ln \nu^*_{2sx} - [E_{1s}(x) + E_{2s}(x)]\nu^*_{2sx}\}.$$

Therefore,

$$Q^* - Q_0 = -2 \ln(\Lambda^*_0/\Lambda^*) + 2 \ln(\hat{\Lambda}_0/\hat{\Lambda})$$

$$= Q^*_0 + \sum_{s=1}^{2} \sum_{x=1}^{16} \{[M_{1s}(x) + M_{2s}(x)] \ln \frac{M_{1s}(x) + M_{2s}(x)}{E_{1s}(x) + E_{2s}(x)}$$

$$- [M_{1s}(x) \ln \frac{M_{1s}(x)}{E_{1s}(x)} + M_{2s}(x) \ln \frac{M_{2s}(x)}{E_{2s}(x)}]\}.$$

Evidently, $Q^* - Q_0$ does not necessarily coincide with Q^*_0.

APPENDIX E. OCCURRENCES AND EXPOSURES

Table 9. Number of marriages recorded for parity 1 women in consensual unions in the Danish fertility survey of 1975. By cohort, starting age, and ordinal months since first birth.

Ordinal month	Ages 15-19 at start of cohabitation: Cohort born in 1926-40	1941-45	1946-50	1951-55	Ages 20-24 at start of cohabitation: Cohort born in 1926-40	1941-45	1946-50	1951-55
0	0	0	0	0	0	0	0	0
1	2	2	1	0	1	0	0	0
2	2	3	3	2	1	2	2	4
3	2	3	2	2	1	1	3	0
4	1	0	2	0	0	1	0	0
5	2	1	2	2	0	0	0	0
6	0	3	0	0	0	0	0	0
7	0	0	2	0	0	1	0	1
8	0	1	1	0	0	1	0	0
9	0	2	1	0	0	0	1	0
10	0	1	1	0	0	0	1	0
11	2	0	0	0	0	0	0	0
12	1	1	0	0	0	0	1	1
13	0	2	0	0	0	0	2	0
14	0	1	0	0	0	1	0	0
15	0	0	1	0	0	1	0	0
16	1	0	0	0	0	0	0	0

Table 10. Woman-months of exposure to the risk of marriage corresponding to the occurrences of Table 9.

Ordinal month	Ages 15-19 at start of cohabitation: Cohort born in 1926-40	1941-45	1946-50	1951-55	Ages 20-24 at start of cohabitation: Cohort born in 1926-40	1941-45	1946-50	1951-55
1	32½	26	22½	11	11	24	15	11
2	28½	23½	20½	9	9	23	14	9
3	25	20½	18	7	7	21½	11½	7
4	23	19	16	6	6	20½	10	7
5	20½	18½	14	5	6	20	10	7
6	19	16½	13	4	6	20	10	7
7	19	14½	12	4	6	19½	10	6½
8	19	13½	10½	4	5½	18½	10	6
9	18½	12	9½	4	5	17½	9½	6
10	18	10½	8½	4	4½	17	8½	6
11	17	10	8	4	3½	17	8	5
12	15½	9½	8	4	3	16½	7½	4½
13	14½	8	8	4	3	16	6	4
14	13	6½	8	4	3	15½	5	4
15	11½	6	7½	4	2½	14	5	3½
16	10½	6	7	4	2	13	5	2½

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